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The distances from the plane, with regard to sign, of the vertices $(1, 1, 1)$, $(1, -1, 1)$, $(-1, 1, 1)$, $(1, 1, -1)$, $(-1, -1, 1)$, $(1, -1, -1)$, $(-1, 1, -1)$, $(-1, -1, -1)$ are $l + m + n - p$, $l - m + n - p$, etc.; and these are in descending order, except the two middle ones, which partake of the property only when $l + m > n$, that is when $\theta > \tan^{-1} (\cos \phi + \sin \phi)^{-1}$.

The plane will cut the cube when $l + m + n > p > -l - m - n$, and will cut it in a hexagon when $l + m - n > p > -l - m + n$. Hence, the required chance is

$$\frac{\displaystyle\int_0^{\pi/4} \int_{\tan^{-1}(\cos \phi + \sin \phi)^{-1}}^{\tan^{-1} \sec \phi} \int_{\cos \theta - \sin \theta(\cos \phi + \sin \phi)}^{-\cos \theta + \sin \theta(\cos \phi + \sin \phi)} \sin \theta \, dp \, d\theta \, d\phi}{\displaystyle\int_0^{\pi/4} \int_0^{\tan^{-1} \sec \phi} \int_{-\cos \theta - \sin \theta(\cos \phi + \sin \phi)}^{\cos \theta + \sin \theta(\cos \phi + \sin \phi)} \sin \theta \, dp \, d\theta \, d\phi}.$$

The integrations for p and θ having been effected, the inverse tangents in ϕ may be removed by integration by parts, after which the substitution $u = \tan \phi$ reduces the integrands to rational functions of u. The resulting value for the probability is $2/\sqrt{3} - (4\sqrt{2}/\pi) \tan^{-1} 1/\sqrt{2}$, or $.0465\cdots$.

2693 [April, 1918]. Proposed by W. F. HARLOW, Portland, Oregon.

A cow is tethered with a rope, length l, to a peg on the opposite side of a wall, height h, the peg being at a distance a from the wall. Find the area over which the cow can graze.

SOLUTION BY C. F. GUMMER, Queen's University, Kingston, Ont.

Let the bottom of the wall be $x = z = 0$; the top $x = 0$, $z = h$; and the peg $(-a, 0, 0)$. The cow being at $(x, y, 0)$, the lengths of the two parts of the rope being u and v, and the point at which the rope crosses the wall being $(0, t, h)$, we must have, if the rope is taut,

$$u^2 = a^2 + h^2 + t^2, \qquad v^2 = x^2 + (t - y)^2 + h^2,$$

and $u + v = l$.

The solution of these equations in t, u, v is

$$t = \frac{yp \pm lr}{2(l^2 - y^2)}, \qquad u = \frac{lp \pm yr}{2(l^2 - y^2)}, \qquad v = \frac{lq \mp yr}{2(l^2 - y^2)},$$

where $p = a^2 + l^2 - x^2 - y^2$, $q = l^2 + x^2 - a^2 - y^2$, and $r = \sqrt{p^2 - 4(a^2 + h^2)(l^2 - y^2)}$.

We require the conditions that u and v may be positive and t real.

From the obtuse triangle $(-a, 0, 0)$, $(0, 0, h)$, (x, y, h) it is clearly necessary that

$$l^2 > a^2 + h^2 + x^2 + y^2. \tag{1}$$

Other necessary conditions are that $p^2 \geqq 4(a^2 + h^2)(l^2 - y^2)$, and (since $l > |y|$ and $|p| > r$) that $p > 0$. From these two it follows that

$$x^2 \leqq l^2 + a^2 - y^2 - 2\sqrt{a^2 + h^2}\,\sqrt{l^2 - y^2}, \tag{2}$$

which again implies that $(\sqrt{l^2 - y^2} - \sqrt{a^2 + h^2})^2 \geqq h^2$, that is (with the help of (1)) that

$$y^2 \leqq l^2 - (\sqrt{a^2 + h^2} + h)^2. \tag{3}$$

It remains to prove that (2) and (3) are not only necessary but sufficient. For this we need only observe that they make r real (and positive) $l > |y|$, $p > r$, and q positive. The lower signs will then give a suitable solution in t, u, and v.

The area to be computed is therefore

$$A = 2\int_0^{\sqrt{l^2 - (\sqrt{a^2 + h^2} + h)^2}} \sqrt{a^2 + l^2 - y^2 - 2\sqrt{a^2 + h^2}\sqrt{l^2 - y^2}} \cdot dy.$$

Writing $\sqrt{l^2 - y^2} = u$, $\sqrt{a^2 + h^2} = c$, we get

$$A = 2\int_{c+h}^{l} \frac{u\sqrt{(u - c)^2 - h^2}}{\sqrt{l^2 - u^2}}\, du.$$

To reduce to standard elliptic integrals, we may make

$$u = \frac{\lambda v + \mu}{v + 1},$$

where $(\lambda - c)(\mu - c) = h^2$, $\lambda\mu = l^2$, $\lambda \geqq \mu$. If l is assumed $\geqq c + h$, as is natural, λ and μ are real, unequal, and greater than c. The result of the transformation is

$$A = 2(\lambda - \mu) \int_{\sqrt{\frac{\mu-c}{\lambda-c}}}^{\sqrt{\frac{\mu}{\lambda}}} \frac{\sqrt{(\lambda - c)v^2 - (\mu - c)}}{\sqrt{\mu - \lambda v^2}} \cdot \frac{\lambda v + \mu}{(v+1)^3} dv.$$

The final result will be simplified if we transform again, putting

$$v = \frac{\sqrt{\mu(\mu - c)}}{\sqrt{\mu(\lambda - c) - (\lambda - \mu)cw^2}},$$

which gives

$$A = 2c(\mu - c) \int_0^1 \frac{(3\lambda c + 3\mu c - 2\lambda\mu - 4c^2)\mu^2 + (\lambda\mu + 5\mu c - 3\mu^2 - 3\lambda c)\mu c w^2 + (\lambda - \mu)\mu c^2 w^4}{(\mu - cw^2)^3 \sqrt{1 - w^2}\sqrt{\mu(\lambda - c) - (\lambda - \mu)cw^2}} \cdot w^2 dw$$

$$+ 2c(\mu - c)\sqrt{\mu(\mu - c)} \int_0^1 \frac{(\lambda + \mu - 4c)\mu - (\lambda - 3\mu)cw^2}{(\mu - cw^2)^3\sqrt{1 - w^2}} \cdot w^2 dw.$$

These integrals are respectively elliptic and circular. On making the necessary reductions, we find, after much calculation,

$$A = c(\mu - c)\sqrt{\frac{\mu}{\lambda - c}} F\left(\sqrt{\frac{(\lambda - \mu)c}{\mu(\lambda - c)}}, \frac{\pi}{2}\right) - c\sqrt{\mu(\lambda - c)} E\left(\sqrt{\frac{(\lambda - \mu)c}{\mu(\lambda - c)}}, \frac{\pi}{2}\right)$$

$$- \frac{c(\mu - c)(\lambda + \mu - c)}{\sqrt{\mu(\lambda - c)}} \Pi\left(-\frac{c}{\mu}, \sqrt{\frac{(\lambda - \mu)c}{\mu(\lambda - c)}}, \frac{\pi}{2}\right) + \frac{\pi}{2} \cdot c(\lambda + \mu - c).$$

2697 [April, 1918]. Proposed by H. S. UHLER, Yale University.

Show how to reduce the left-hand members of the following identities to their respective right members:

$$\sin^2 (x + \tfrac{1}{2}y) - \sin (x + \tfrac{3}{2}y) \sin (x - \tfrac{1}{2}y) = \sin^2 y, \qquad (1)$$

$$\sin (x + y) \sin (x + \tfrac{1}{2}y) - \sin x \sin (x + \tfrac{3}{2}y) = \sin \tfrac{1}{2}y \sin y, \qquad (2)$$

$$\sin x \sin (x + \tfrac{1}{2}y) - \sin (x - \tfrac{1}{2}y) \sin (x + y) = \sin \tfrac{1}{2}y \sin y. \qquad (3)$$

SOLUTION BY POLYCARP HANSEN, St. John's University, Collegeville, Minn.

The terms of the left-hand members can be expressed as follows:

(1)
$$\sin^2 (x + \tfrac{1}{2}y) - \sin (x + \tfrac{3}{2}y) \sin (x - \tfrac{1}{2}y)$$
$$= \frac{1 - \cos (2x + y)}{2} + \frac{1}{2}[\cos (2x + y) - \cos 2y] = \frac{1 - \cos 2y}{2} = \sin^2 y.$$

(2)
$$\sin (x + y) \sin (x + \tfrac{1}{2}y) - \sin x \sin (x + \tfrac{3}{2}y) = - \tfrac{1}{2}[\cos (2x + \tfrac{3}{2}y) - \cos \tfrac{1}{2}y]$$
$$+ \tfrac{1}{2}[\cos (2x + \tfrac{3}{2}y) - \cos (- \tfrac{3}{2}y)] = \tfrac{1}{2}[\cos \tfrac{1}{2}y - \cos (- \tfrac{3}{2}y)] = \sin \tfrac{1}{2}y \sin y.$$

(3)
$$\sin x \sin (x + \tfrac{1}{2}y) - \sin (x - \tfrac{1}{2}y) \sin (x + y) = - \tfrac{1}{2}[\cos (2x + \tfrac{1}{2}y) - \cos (- \tfrac{1}{2}y)]$$
$$+ \tfrac{1}{2}[\cos (2x + \tfrac{1}{2}y) - \cos (- \tfrac{3}{2}y)] = \sin \tfrac{1}{2}y \sin y.$$

Also solved by R. B. WILDERMUTH, JEROME J. JULIAN, KATHERINE S. ARNOLD, R. M. MATHEWS, H. L. OLSON, H. E. CARLSON, A. T. DINEEN, R. C. COLWELL, and ROGER A. JOHNSON.

2698 [April, 1918]. Proposed by WARREN WEAVER, Throop College of Technology, Pasadena, California.

An urn contains N balls, numbered from 1 to N. Of these n are drawn out and are arranged linearly according to the numbers on each. A certain ball is observed to be the kth in this line. What is the most probable number written on this ball?